

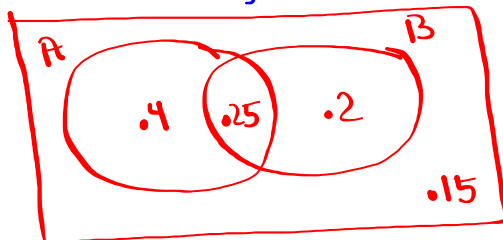
**Math 110**  
**Winter 2021**  
**Lecture 11**



Class QZ 6

Given:  $P(A) = .65$      $P(B) = .45$      $P(A \text{ and } B) = .25$

1) Venn Diagram



2)  $P(\bar{B})$

$$= 1 - .45 = \boxed{.55}$$

3)  $P(A \text{ or } B)$

$$= .4 + .25 + .2$$

$$= \boxed{.85}$$

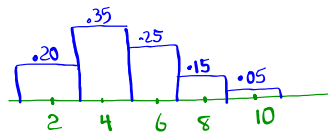
Consider the following chart

$x$	$P(x)$
2	.20
4	.35
6	.25
8	.15
10	.05

1) Find  $P(x=2)$

$$= 1 - [.35 + .25 + .15 + .05] = 1 - .8 = .2$$

2) Draw Prob. dist. histogram



3) Find  $\mu$  &  $\sigma$

$x \rightarrow L1$  STAT CALC 1-VAR stats L1 & L2  
 $P(x) \rightarrow L2$   $\mu = 5$   $\sigma = 2.236$

4) Find exact value of  $\sigma^2$

VAR S [5:] [4:]  $x^2$  [Math] [1:] [Enter]

$\mu = 5.0$   $\sigma = 2.2$   $\sigma^2 = 5$

5) Round  $\mu$  &  $\sigma$  to 1-decimal, then find

its usual Range  $\mu \pm 2\sigma = 5.0 \pm 2(2.2)$   
 "95% Range"  $= 5.0 \pm 4.4 \Rightarrow .6 \text{ to } 9.4$

I sold 50 tickets, \$20 each.

one ticket is drawn, the winner get a tablet worth \$250. Find expected value per ticket.

\$Net	$P(\$Net)$
20 - 250	$\frac{1}{50}$
20 - 0	$\frac{49}{50}$

$\$Net \rightarrow L1$

$P(\$Net) \rightarrow L2$

Expected Value =  $\mu = \bar{x} = \$15$

$\$15/\text{Ticket}$

You are having a Surgery.

You buy insurance for \$500. If you don't make it, insurance company pays \$100,000 to your family.

Prob. that you don't make it out of the surgery is .5%. Find expected value per policy sold.

\$Net | P(\$Net)

500-100,000 | .5% = .005

500 - 0 | 99.5% = .995

\$Net → L1

P(\$Net) → L2

E.V. =  $\mu = \bar{x} = \boxed{0}$

No Profit, No Loss

A box has 3 Dimes & 12 Nickels.

Shake it to get 2 coins.

DD → 20¢  $P(20¢) = \frac{3}{15} \cdot \frac{2}{14} = \frac{6}{210}$

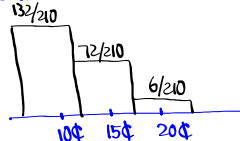
DN → 15¢  $P(15¢) = 1 - \left[ \frac{6}{210} + \frac{132}{210} \right] = \frac{72}{210}$

ND

NN → 10¢  $P(10¢) = \frac{12}{15} \cdot \frac{11}{14} = \frac{132}{210}$

Collect ¢	P(Collect ¢)
10¢	$\frac{132}{210}$
15¢	$\frac{72}{210}$
20¢	$\frac{6}{210}$

Draw Prob. Dist. histogram



Collect ¢ → L1

Σind

P(Collect ¢) → L2

$\mu = 12$

$\sigma = 2.726$   $\sigma^2 = \frac{52}{7}$

Round  $\mu$  &  $\sigma$  to a whole #, Σind **68% Range**

$\mu = 12$  &  $\sigma \approx 3$

$\mu \pm \sigma$

$= 12 \pm 3$

$\Rightarrow \boxed{9 \text{ to } 15}$

Exam II  
Next Monday

## Binomial Prob. Dist.

- 1)  $n$  independent events
- 2) Each event has only two outcomes.  
     • Success                      • Failure
- 3)  $P(\text{Success}) = p$                $P(\text{Failure}) = q$   
      $p + q = 1$                        $q = 1 - p$   
      $p$  &  $q$  remain unchanged for all  $n$  events
- 4)  $x$  is # of Successes

$$P(x) = {}^n C_x \cdot p^x \cdot q^{n-x}$$

Given a binomial Prob. dist with  $n=25$  and  
 $p=.6$

Find  $P(x=10) = {}^{25} C_{10} \cdot (.6)^{10} \cdot (.4)^{15}$   
 $= \boxed{.021}$

$$P(x) = {}^n C_x \cdot p^x \cdot q^{n-x}$$

25 math Prob  $nCr$  10 \* .6  $\boxed{\wedge}$  10 \* .4  $\boxed{\wedge}$  15  
 $\boxed{\text{Enter}}$



You are making random guesses on a True/False exam with 40 questions.

$$n=40, P=.5, q=.5$$

$$P(25 \text{ correct ans}) =$$

$$P(X=25) = {}_{40}C_{25} * (.5)^{25} * (.5)^{15} = \boxed{.037}$$

2nd VARS ↓ ↓ ... binom PDF

Menu

Trials: 40

P: .5

X-value: 25

Paste Enter

No Menu

binompdf(40, .5, 25)

n

P

x

Enter

= .037

A loaded coin is tossed 80 times.

Prob. of landing tails is .6

$$P(\text{exactly 50 tails}) = P(X=50) \\ = \text{binompdf}(80, .6, 50) \\ = \boxed{.083}$$

$$P(\text{at most 55 tails}) =$$

$$P(X \leq 55) = \text{binomcdf}(80, .6, 55) = \boxed{.958}$$

You are taking an exam with 50 multiple-choice questions.

Each question has 4 choices. Only one correct choice.

You are making random guesses.

$$n = 50 \quad p = \frac{1}{4} = .25 \quad q = \frac{3}{4} = .75$$

$$P(\text{exactly 15 correct ans}) = P(X=15) \\ = \text{binompdf}(50, .25, 15)$$

$$P(\text{fewer than 20 correct ans}) = \boxed{.089}$$

$$P(X < 20) = P(X \leq 19) = \text{binomcdf}(50, .25, 19) \\ = \boxed{.986}$$

UPS says prob. that any package is on time is .9.

80 packages were randomly selected.

$$1) P(\text{exactly 70 are on time}) =$$

$$P(X=70) = \text{binompdf}(80, .9, 70) = \boxed{.103}$$

$$2) P(\text{at most 75 are on time}) =$$

$$P(X \leq 75) = \text{binomcdf}(80, .9, 75) = \boxed{.912}$$

$$3) P(\text{at least 65 on time}) =$$

$$P(X \geq 65) = 1 - P(X \leq 64) = 1 - \text{binomcdf}(80, .9, 64) \\ = \boxed{.995}$$

100 Newborn babies were randomly selected.

$$n=100 \quad p=.5 \quad q=.5$$

$$\begin{aligned} P(\text{exactly 40 boys}) &= P(X=40) \\ &= \text{binom pdf}(100, .5, 40) \\ &= \boxed{.011} \end{aligned}$$

$$\begin{aligned} P(\text{Fewer than 60 girls}) &= P(X < 60) \\ &= P(X \leq 59) \\ &= \text{binomcdf}(100, .5, 59) \\ &= \boxed{.972} \end{aligned}$$

$$\begin{aligned} P(X > 45) &= P(X \geq 46) \\ &= 1 - P(X \leq 45) = 1 - \text{binomcdf}(100, .5, 45) \\ &= \boxed{.816} \end{aligned}$$

~~45~~ 46 ✓

$$P(X=a) = \text{binom pdf}(n, p, a)$$

$$P(X \leq a) = \text{binomcdf}(n, p, a)$$

$$P(X \geq a) = 1 - \text{binomcdf}(n, p, a-1)$$

$$P(a \leq X \leq b) = \text{binomcdf}(n, p, b) - \text{binomcdf}(n, p, a-1)$$

Reduce by 1

Consider a binomial Prob. dist with  $n=120$ , and  $p=.8$

$$\begin{aligned} \text{Find } P(95 \leq X \leq 110) \\ &= \text{binomcdf}(120, .8, 110) - \text{binomcdf}(120, .8, 94) = \boxed{.641} \end{aligned}$$

Prob. of full recovery from certain surgery is .85. 145 of these surgeries are randomly selected. Find the prob. that from 110 to 130 of them have full recovery.

$$n=145, p=.85$$

$$P(110 \leq X \leq 130)$$

Reduce by 1

$$= \text{binomcdf}(145, .85, 130) - \text{binomcdf}(145, .85, 109)$$

$$= \boxed{.958}$$

Binomial Prob. Dist:

Mean

$\mu$

$$\boxed{\mu = np}$$

Variance

$\sigma^2$

$$\boxed{\sigma^2 = npq}$$

Standard Deviation  $\sigma$

$$\sigma = \sqrt{\sigma^2}$$

Toss a fair coin 400 times.

$$n=400 \quad p=.5 \quad q=.5$$

$$\mu=np \quad \mu=400(.5) \quad \boxed{\mu=200}$$

$$\sigma^2=npq \quad \sigma^2=400(.5)(.5) \quad \boxed{\sigma^2=100}$$

$$\sigma=\sqrt{\sigma^2}=\sqrt{100}=\boxed{10}$$

Usual Range

$$\begin{aligned} \mu \pm 2\sigma \\ = 200 \pm 2(10) \\ = 200 \pm 20 \end{aligned}$$

$$\Rightarrow \boxed{180 \text{ to } 220}$$

You are making random guesses on a test with 200 questions. Each question has 5 choices with only one correct choice. Success is to get a correct answer.

$$1) n=200 \quad 2) p=\frac{1}{5}=.2 \quad 3) q=\frac{4}{5}=.8$$

$$\begin{aligned} 4) \mu=np &= 200(.2) = \boxed{40} \\ 5) \sigma^2=npq &= 200(.2)(.8) = \boxed{32} \\ 6) \sigma=\sqrt{\sigma^2} &= \sqrt{32} = \boxed{5.657} \end{aligned}$$

Round  $\mu$  &  $\sigma$  to a whole #, find  $\mu=40, \sigma=6$

7) Usual range

$$95\% \text{ Range } \mu \pm 2\sigma = 40 \pm 2(6) = 40 \pm 12 \Rightarrow \boxed{28 \text{ to } 52}$$

8)  $P(\text{between } 28 \text{ and } 52 \text{ correct ans, inclusive})$

$$\begin{aligned} = P(28 \leq X \leq 52) &= \text{binomcdf}(200, .2, 52) - \\ &\quad \text{binomcdf}(200, .2, 27) = \boxed{.973} \end{aligned}$$

Reduce by 1